Inverse Theorem

$F'(a) = \frac{1}{f'(f^{-1}(a))}$

*F' shows if a function is increasing or decreasing

$\frac{1}{csc} = \sin$ $\quad \frac{1}{cos} = \sec$ $\quad \frac{\sin}{\cos} = \tan$

$\frac{1}{sec} = \cos$ $\quad \frac{1}{sin} = \csc$ $\quad \frac{\cos}{\sin} = \cot an$

$\int \ln(x) = x\ln(x) - x + C$

$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$

	0	Π/6	Π/4	Π/3	Π/2
	0	30	45	60	90
sin	0	1/2	√2/2	√3/2	√4/2
cos	√4/2	√3/2	√2/2	1/2	0
tan	0	√3/3	1	√3	undef

DERIVATIVES / INTEGRALS

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x\ln(a)$

$\frac{d}{dx}(\log_a x) = \frac{1}{x\ln(a)}$

$\frac{d}{dx}\ln|x| = |\frac{1}{x}|$

$\int \frac{1}{x}dx = \ln|x| + C$

INVERSE FUNCTIONS

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

$\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$

$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}\csc^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$

HYPERBOLICS

* most common

* $\sinh x = \frac{e^x - e^{-x}}{2}$

* $\cosh x = \frac{e^x + e^{-x}}{2}$

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\text{csch}\, x = \frac{1}{\sinh}$

$\text{sech}\, x = \frac{1}{\cosh}$

$\coth x = \frac{1}{\tanh} = \frac{\cosh}{\sinh}$

$\frac{d}{dx}\sinh x = \cosh x$

$\frac{d}{dx}\tanh x = \text{sech}^2 x$

$\frac{d}{dx}\text{sech}\, x = -\text{sech}\, x\tanh x$

$\frac{d}{dx}\coth x = -\text{csch}^2 x$

$\frac{d}{dx}\cosh x = \sinh x$

$\frac{d}{dx}\text{csch}\, x = -\text{csch}\, x\coth x$

$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + c$

$\int -\frac{1}{\sqrt{1-x^2}}dx = \cos^{-1}x + c$

$\int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

$\int -\frac{1}{x\sqrt{x^2-1}}dx = \csc^{-1}x + c$

$\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}x + c$

$\int -\frac{1}{1+x^2}dx = \cot^{-1}x + c$

IDENTITIES

$\sin 2x = 2\sin x\cos x$

$\sin^2 x + \cos^2 x = 1$ (or) $\sin^2 x = 1 - \cos^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\frac{1}{\cos^2 x} = \sec^2 x$

RULES

odd power on cos x => u = sin x

odd power on sin x => u = cos x

odd power on tan x => u = sec x

both even: sin 2x = 2sin x cos x

METHODS OF INTEGRATION

BY PARTS

ILATE $\quad UV - \int VDU$

(inverse, logarithm, algebraic, trigonometric, exponential)

COMPARISON THEOREM

p > 1 => function converges

p < 1 => function diverges

PARTIAL FRACTIONS

$x^2 \Rightarrow \frac{A}{x} + \frac{B}{x^2}$

$(x+1)^2 \Rightarrow \frac{A}{x+1} + \frac{B}{(x+1)^2}$

$(x^2+x+1)^2 \Rightarrow \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2}$

TRIG INTEGRALS

(of type)

$\int \sin^m x + \cos^n x\, dx$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

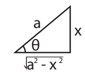
$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\csc x = -\csc x\cot x$

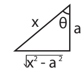
$\frac{d}{dx}\sec x = \sec x\tan x$

$\frac{d}{dx}\cot x = -\csc^2 x$

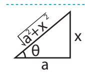
TRIG SUBSTITUTION



$\sin\theta = \frac{x}{a} \Rightarrow$ $x = a\sin\theta$, $dx = a\cos\theta\, d\theta$, $\cos\theta = \frac{\sqrt{a^2-x^2}}{a}$, $\sqrt{a^2-x^2} = a\cos\theta$



$\sec\theta = \frac{x}{a} \Rightarrow$ $x = a\sec\theta$, $dx = a\sec\theta\tan\theta\, d\theta$, $\tan\theta = \frac{\sqrt{x^2-a^2}}{a}$, $\sqrt{x^2-a^2} = a\tan\theta$



$\tan\theta = \frac{x}{a} \Rightarrow$ $x = a\tan\theta$, $dx = a\sec^2\theta\, d\theta$, $\sec\theta = \frac{\sqrt{x^2+a^2}}{a}$, $\sqrt{x^2+a^2} = a\sec\theta$

SUMMARY OF SERIES CONVERGENCE AND DIVERGENCE TESTS

DIVERGENCE TEST

$\sum a_n$ — Diverges if $\lim_{n\to\infty} a_n \neq 0$ — Inconclusive if $\lim_{n\to\infty} a_n = 0$

GEOMETRIC SERIES

$\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=0}^{\infty} ar^n$ — Converges $s = \frac{a}{1-r}$ if $|r| < 1$; Diverges if $|r| \geq 1$ — Useful for doing comparison tests on series with general term similar to ar^{n-1}

P-SERIES

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ — Converges if p > 1; Diverges if p ≤ 1 — Useful for doing comparison tests on series with general term similar to $\frac{1}{n^p}$

INTEGRAL

$\sum_{n=1}^{\infty} a_n$, $f(n) = a_n$ — Converges if $\int_1^\infty f(x)dx$ converges; Diverges if $\int_1^\infty f(x)dx$ diverges — the function f(x) must be continuous and (eventually) positive and decreasing ... and something you don't mind integrating.

DIRECT COMPARISON

$\sum_{n=1}^{\infty} a_n$ "like" $\sum_{n=1}^{\infty} b_n$ — if $\sum b_n$ converges and $a_n \leq b_n$ then $\sum a_n$ converges; if $\sum b_n$ diverges and $a_n \geq b_n$ then $\sum a_n$ diverges — To decide which series $\sum b_n$ to compare to, consider terms of $\sum a_n$ that have the greatest effect

LIMIT COMPARISON

$\sum_{n=1}^{\infty} a_n$ "like" $\sum_{n=1}^{\infty} b_n$ — if $\lim_{n\to\infty}\frac{a}{b} = L$, with L > 0 then both series converge or both diverge. If L=0 and $\sum b_n$ converges, so does $\sum a_n$. If $L = \infty$, and $\sum b_n$ diverges, so does $\sum a_n$. — If the inequality needed for direct comparison doesn't work, try limit comparison.

ALTERNATING SERIES *(Tells only if convergent or divergent)*

$\sum_{n=1}^{\infty}(-1)^n b_n$, $b_n > 0$ — Converges if: b_n is eventually *decreasing* and $\lim_{n\to\infty} b_n = 0$ — Only applies to alt. series! It's ok if the exponent on the -1 is n+1 or n-1.

RATIO TEST

$\sum_{n=1}^{\infty} a_n$ — if $\lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = L$ (or ∞) the series: converges **absolutely** if L<1; diverges if L>1 (or is ∞) — This test gives no information if L=1. This test works well if *nth* powers are involved.

ABSOLUTE CONVERGENCE TEST

$\sum_{n=1}^{\infty} a_n$ — if $\sum_{n=1}^{\infty}|a_n|$ converges, so does $\sum_{n=1}^{\infty} a_n$ — Useful for series that have both positive and negative terms, but aren't necessarily alternating.

IMPORTANT MACLAURIN SERIES

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + ...$ $\quad (-1,1)$

$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} ...$ $\quad (-\infty,\infty)$

$\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{1!} + \frac{x^5}{2!} - \frac{x^7}{3!} + ...$ $\quad (-\infty,\infty)$

$\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + ...$ $\quad (-\infty,\infty)$

$\tan^{-1}x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$ $\quad [-1,1]$

POLAR CURVES

$\frac{dy}{d\theta} = r'\sin\theta + r\cos\theta$ $\quad \frac{dx}{d\theta} = r'\cos\theta - r\sin\theta$

Polar to Cartesian: $x = r\cos\theta$ $\quad y = r\sin\theta$

Cartesian to Polar: $r^2 = x^2 + y^2$ $\quad \tan\theta = \frac{y}{x}$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dy}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

Areas: $A = \int_a^b \frac{1}{2}r^2 d\theta$ OR $A = \int_a^b \frac{1}{2}[f(\theta)]^2 d\theta$

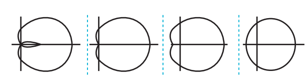
Arclength: $L = \int_a^b \sqrt{r^2 + [\frac{dr}{d\theta}]^2}\, d\theta$

Limacons

$r = a \pm b\sin\theta$

a > 0 , b > 0

(plot 4 points!)



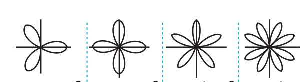
$\frac{a}{b} < 1$ $\quad \frac{a}{b} = 1$ $\quad 1 < \frac{a}{b} < 2$ $\quad \frac{a}{b} \geq 2$

Rose Curves

n ≥ 2

length of each petal is 'a'

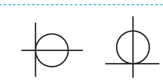
if n is odd, curve is traced twice on 0 ≤ θ < 2Π



r= a cos nθ, n petals, n odd — r=a cos nθ, 2n petals, n even — r= a sin nθ, n petals, n odd — r= a sin nθ, 2n petals, n even

Circles

radius is $\frac{a}{2}$



r= a cos θ $\quad$ r= a sin θ

DOUBLE ANGLE FORMULA

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

HALF ANGLE FORMULA

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

TABLE OF INTEGRALS (basic forms)

$\int u\,dv = uv - \int v\,du$

$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$

$\int \frac{du}{u} = \ln|u| + C$

$\int e^u du = e^u + C$

$\int a^u du = \frac{a^u}{\ln a} + C$

$\int \sin u\,du = -\cos u + C$

$\int \cos u\,du = \sin u + C$

$\int \sec^2 u\,du = \tan u + C$

$\int \csc^2 u\,du = -\cot u + C$

$\int \sec u\tan u\,du = \sec u + C$

$\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

$\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

TABLE OF INTEGRALS (Inverse Trig Forms)

$\int \sin^{-1}u\,du = u\sin^{-1}u + \sqrt{1-u^2} + C$

$\int \cos^{-1}u\,du = u\cos^{-1}u - \sqrt{1-u^2} + C$

$\int \tan^{-1}u\,du = u\tan^{-1}u - \frac{1}{2}\ln(1+u^2) + C$

$\int \frac{du}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

RULES OF $\lim_{n\to\infty} r^n$

$\lim_{n\to\infty} r^n$

if r > 1 $\{r^n\}$ diverges

if r = 1 $\{r^n\}$ converges to 1

if 0 < r < 1 $\{r^n\}$ converges to 0

if r = 0 $\{r^n\}$ converges to 0

if -1 < r < 0 $\{r^n\}$ converges to 0

if r ≤ -1 $\{r^n\}$ diverges

L'HOPITAL RULE INDETERMINATE FORMS

products: $0 * \pm\infty$

difference: $\infty - \infty$

quotient: $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$

powers: 0^0 or ∞^0 or 1^∞

GENERAL FORMULAS OF DERIVATIVES

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule)

$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)

$\frac{d}{dx}[cf(x)] = cf'(x)$

$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Quotient Rule)

$\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)

TAYLOR SERIES AND TAYLOR POLINOMIAL AT VALUE 'a'

Step 1: Build Chart $\quad$ Step 2: Find Pattern

$\sum_{n=0}^{\infty} C_n(x-a)^{(n)}$

n	$f^{(n)}(x)$	$f^{(n)}(a)$	$\frac{f^{(n)}(a)}{n!}$
0	$f^{(0)}(x)$	$f^{(0)}(a)$	$\frac{f^{(0)}(a)}{n!}$
1	$f^{(1)}(x)$	$f^{(1)}(a)$	$\frac{f^{(1)}(a)}{n!}$
2	$f^{(2)}(x)$	$f^{(2)}(a)$	$\frac{f^{(2)}(a)}{n!}$
3	$f^{(3)}(x)$	$f^{(3)}(a)$	$\frac{f^{(3)}(a)}{n!}$

Taylor Polynomial Sample

$P(x) = f(a) + \frac{f'(a)}{n!}(x-a) + \frac{f''(a)}{n!}(x-a)^2 + \frac{f'''(a)}{n!}(x-a)^3 + ... \frac{f^{(n)}(a)}{n!}(x-a)^n$

STANDARD EQUATIONS

Standard Arc Length = $\int_a^b \sqrt{1 + (f'(x))^2}\, dx$

Surface Area = $\int_a^b 2\Pi(\text{radius})\sqrt{1 + (f'(x))^2}\, dx$

PARAMETRIC EQUATIONS

Arc Length = $\int_\alpha^\beta \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$

Surface Area = $\int_a^b 2\Pi(\text{radius})\sqrt{(f'(t))^2 + (g'(t))^2}\, dx$

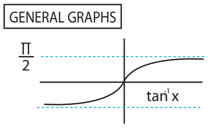
Area = $\int_\alpha^\beta g(t)\,(f'(t))\, dt$

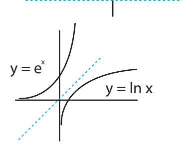
(Remember to check limits of integration)

First Derivative $\quad$ Second Derivative

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $\quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$

GENERAL GRAPHS





RADIUS / INTERVAL OF CONVERGENCE

(After limit has been taken)

$|X + a| * \infty < 1$ when $r = 0$: $I = \{-a\}$

$|X + a| * 0 < 1$ when $r = \infty$: $I = (-\infty, \infty)$

All other cases check endpoints